

Fluctuations in viscous fingering

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Our experiments on viscous (Saffman-Taylor) fingering in Hele-Shaw channels reveal finger width fluctuations that were not observed in previous experiments, which had lower aspect ratios and higher capillary numbers Ca . These fluctuations intermittently narrow the finger from its expected width. The magnitude of these fluctuations is described by a power law, $Ca^{-0.64}$, which holds for all aspect ratios studied up to the onset of tip instabilities. Further, for large aspect ratios, the mean finger width exhibits a maximum as Ca is decreased instead of the predicted monotonic increase.

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When a less viscous fluid displaces a more viscous fluid in a Hele-Shaw channel (a quasi-two-dimensional geometry in which the width w is much greater than the channel thickness b), the interface between the fluids forms a pattern of growing “fingers.” A single finger forms at low flow rates; more complex branched patterns evolve at high flow rates. This phenomenon is the prototypical example of moving interface problems, such as dendritic growth and flame propagation, and thus continues to receive attention for the insight it provides into these important problems [1]. The problem was studied first by Saffman and Taylor [2], who injected air into oil in a Hele-Shaw cell and observed the formation of a single, steadily moving finger whose width decreased monotonically to $1/2$ of the channel width as the finger speed was increased. In subsequent experimental [3], numerical [4], and theoretical [5] work, the ratio of finger width to channel width, λ , was found to depend on a modified capillary number, $1/B = 12(w/b)^2 Ca$, which combines the aspect ratio, w/b , and the capillary number, $Ca = \mu V / \sigma$, where μ is the dynamic viscosity of the liquid, V is the velocity of the tip of the finger, and σ is the surface tension. For large $1/B$ values, a transition to complex patterns of tip-splitting occurs [3,6,7].

Our experiments reveal fluctuations in the width of the evolving viscous fingers that have not been reported in previous experiments [2,3,6] or predicted theoretically. The fluctuations intermittently narrow the fingers from their expected width and are largest at low Ca , falling off as a power law with increasing Ca . Further, at large aspect ratios, the fluctuations are accompanied by substantial departures from the monotonic dependence of the finger width on $1/B$ found previously [3–5]: For aspect ratios $w/b \geq 250$, which were not examined in previous work, we find that the mean finger width no longer scales monotonically with $1/B$; for smaller aspect ratios, our finger width measurements are in accord with previous results. Because large aspect ratios should more closely approach the ideal of two-dimensional flow, our

observations pose a challenge to the assumptions underlying theoretical analyses of viscous fingering.

Experimental Methods. We conducted experiments in a 254 cm long channel formed of 1.9 cm thick glass plates. The spacers between the glass plates were stainless steel strips with thicknesses $b = 0.051$ cm, 0.064 cm, 0.102 cm, or 0.127 cm; the channel width w between the spacers was varied between 19.9 cm and 25.1 cm. Both glass plates were supported at the sides so that they sagged similarly under their own weight (though by less than 0.1% of the gap depth). For aspect ratios under 150, we used a smaller channel of length 102 cm and width 7.4 cm. Interferometric measurements revealed that the root-mean-square variations in gap thickness were typically 0.6% or less in the large channel and 0.8% in the small channel. Mechanical measurements of the bending of the glass due to the imposed pressure gradient revealed that such deflections were typically 0.2% or less; the maximum deflection (2.2%) was measured in the widest channel close to the oil reservoir at the highest flow rates. Experiments were conducted with air penetrating a Dow Corning silicone oil whose surface tension and dynamic viscosity were either $\sigma = 19.6$ dyne/cm, $\mu = 9.21$ cP or $\sigma = 20.6$ dyne/cm, $\mu = 50.8$ cP at laboratory temperature (22°C). The oils wet the glass completely. A uniform flow rate was imposed by withdrawing oil with a syringe pump from a reservoir at one end of the channel; an air reservoir at atmospheric pressure was attached to the other end.

The channel was illuminated from below and images were obtained using a camera and rotating mirror that captured 11 overlapping frames to produce concatenated images of up to $1200 \times 10\,000$ pixels at a resolution of 0.25 mm/pixel. The interfaces were then digitally traced, yielding finger width values accurate to 0.1% in the larger channel and 0.3% in the smaller channel. For each flow rate, up to four time sequences of 20–30 digital interfaces were recorded. Finger widths determined in consecutive sequences agreed within the measurement accuracy. Mean width values agreed within 0.5% for data sets repeated after channel disassembly, cleaning, and reassembly.

For each interface image with a single, well developed finger, we found the instantaneous finger width by averaging measurements taken in a narrow window that was 5% of the channel width beginning 1.2 channel widths behind the tip.

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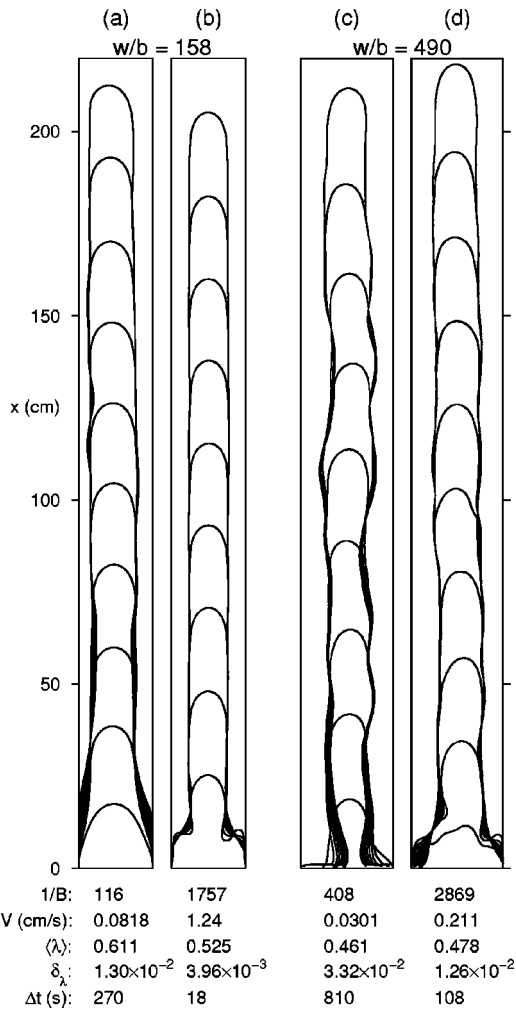


FIG. 1. Finger images recorded at regular time intervals for different values of aspect ratio w/b and modified capillary number $1/B$, with corresponding values of the tip velocity V , the mean finger width $\langle \lambda \rangle$, the rms fluctuation of the finger width δ_λ , and the time Δt between successive curves.

This measurement window was chosen to be small compared to the length scale of the fluctuations (approximately one-third of a channel width); the results were not sensitive to the window's exact location. From each time series of instantaneous widths, we determined the time average $\langle \lambda \rangle$ and the root-mean-square (rms) fluctuation from the mean δ_λ . Each data set was analyzed for flow rates up to the point of tip splitting, beyond which the finger width λ was no longer well defined [8].

Results. Typical interface image sequences are shown for $w/b = 158$ and 490 in Fig. 1. For both aspect ratios the finger width λ fluctuates visibly at low flow velocities [Figs. 1(a) and 1(c)]. In the smaller aspect ratio system the width appears to become steady as the finger velocity is increased [Fig. 1(b)], appearing exactly like the classic “half-width finger” of Saffman and Taylor. However, with sufficient resolution, fluctuations can still be measured for all velocities up to the onset of tip instabilities. In the higher aspect ratio system the width fluctuates visibly for all flow rates [Fig. 1(d)]. The onset of tip instabilities in both cases occurs at

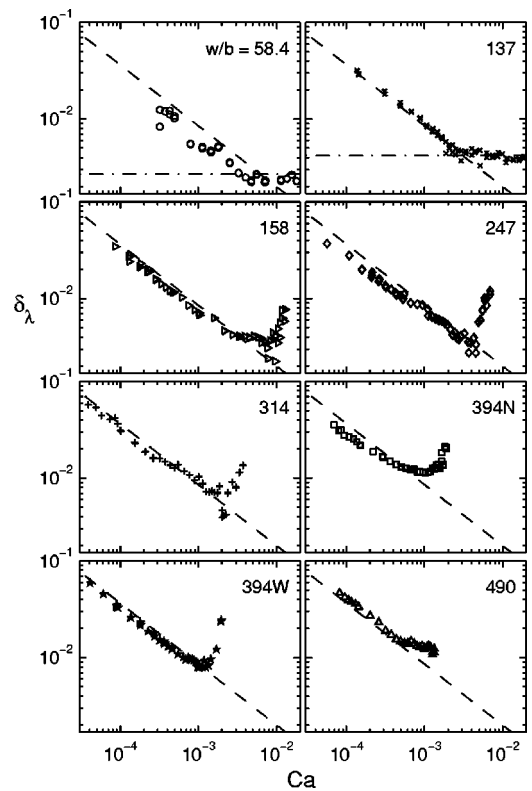


FIG. 2. The rms fluctuation of finger width δ_λ as a function of capillary number Ca , where the dashed lines describe the best fit to all of the data sets within the observed scaling region, $\delta_\lambda = (1.1 \times 10^{-4}) Ca^{-0.64}$. (The data for 394N and 394W have the same aspect ratio but different widths; see caption of Fig. 3.) The horizontal dash-dotted lines in the top two graphs correspond to the limits of measurement accuracy for that channel and geometry; this limit is below the lower edge of the graphs for the other data sets. Fluctuations with $\delta_\lambda \leq 10^{-2}$ are not obvious visually. An upturn in δ_λ occurs at high Ca , signalling the onset of the secondary instabilities in the tips of the fingers.

$1/B \approx 4000$, similar to values seen in previous experiments [3,6].

For all the aspect ratios studied, we find that the rms fluctuation of the finger widths is described by $\delta_\lambda = A(Ca)^\beta$ with $A = (1.1 \pm 0.3) \times 10^{-4}$ and $\beta = -0.64 \pm 0.04$, as Fig. 2 illustrates. We also observe that the instantaneous velocity of the finger tip fluctuates from the average velocity; within the experimental uncertainty these velocity fluctuations scale with Ca in the same manner as the width fluctuations. Interestingly, a dependence of the form $Ca^{-2/3}$ appears frequently in theories of viscous fingering [1,9].

The fluctuations in finger width are accompanied by a substantial deviation from the expected relation between finger width and velocity. Our results for the width of the viscous fingers for high w/b are not described by a single curve as a function of $1/B$ as predicted [5], and the differences between data for different aspect ratios are far greater than those reported previously for low w/b [3]. Figure 3(a) illustrates this, showing the dependence of the mean finger width $\langle \lambda \rangle$ on $1/B$ for values of w/b between 58.4 and 490.

In particular, the mean finger width exhibits a surprising

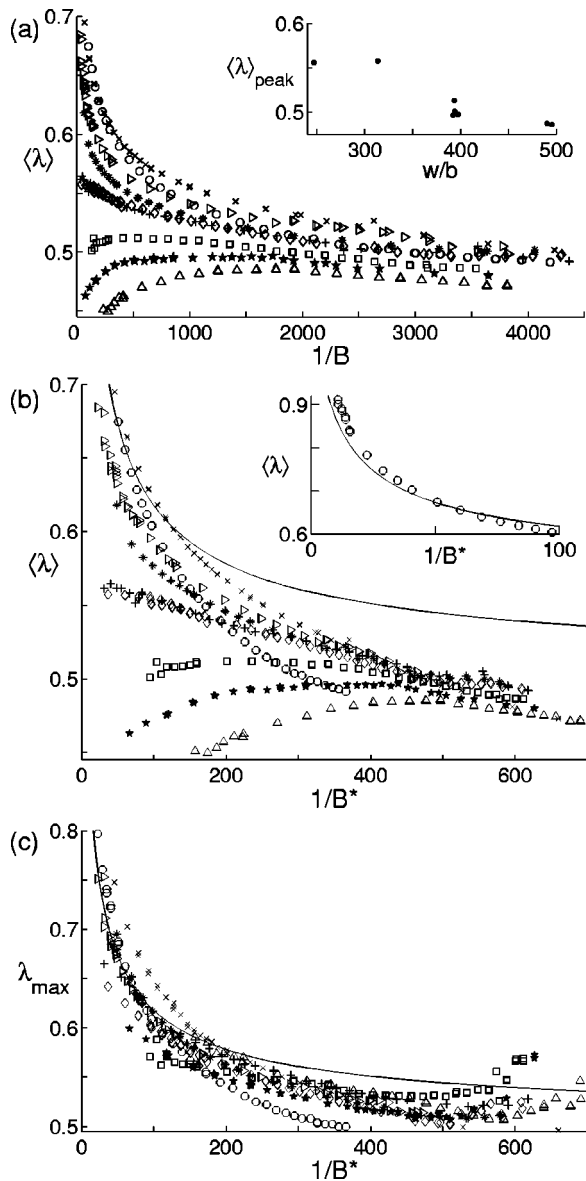


FIG. 3. (a) Mean finger width $\langle \lambda \rangle$ vs $1/B$ for various aspect ratios w/b . Higher aspect ratios give smaller $\langle \lambda \rangle$ values. The large w/b data exhibit a peak width $\langle \lambda \rangle_{\text{peak}}$ as a function of $1/B$; the inset shows dependence of $\langle \lambda \rangle_{\text{peak}}$ on aspect ratio w/b . (b) Mean finger width $\langle \lambda \rangle$ vs $1/B^*$, which includes film wetting corrections. The solid line is the theoretical curve of McLean and Saffman [5]. The inset extends the $w/b=58.4$ curve to lower forcing. (c) Maximum width λ_{max} of the fluctuating finger width observed during evolution as a function of $1/B^*$. Data for $w/b > 137$ collapse roughly onto the same curve, which follows the McLean-Saffman theory more closely than the low aspect ratio results. Aspect ratio symbols: \circ , 58.4 ($w=7.4$ cm); \times , 137 ($w=7.0$ cm); \triangleright , 158 ($w=20.1$ cm); $*$, 214 ($w=22.6$ cm); \diamond , 247 ($w=25.1$ cm); $+$, 314 ($w=19.9$ cm); \square , 394 N ($w=20.0$ cm); $*$, 394 W ($w=25.0$ cm); \triangle , 490 ($w=24.9$ cm).

maximum as the tip velocity is decreased for large aspect ratios [Figs. 3(a) and 3(b)]. The value of $\langle \lambda \rangle$ at the peak, $\langle \lambda \rangle_{\text{peak}}$, is plotted vs w/b in the inset of Fig. 3(a).

To further compare our work with past results, we must account for the effects of the film of oil left behind on the

plates as the finger advances. To do so, we use a correction to the modified capillary number introduced by Tabeling and Libchaber, $1/B^* = (1/B) / [\pi/4 + 1.7\lambda(w/b)(Ca)^{2/3}]$ [9]. Using this correction, they obtained agreement between their experimental measurements of λ and the theoretical predictions of McLean and Saffman [5] for $1/B^* < 100$ ($1/B < 250$); we obtain similar agreement for $w/b=58.4$, as the inset of Fig. 3(b) illustrates. Figure 3(b) shows the dependence of the mean finger width $\langle \lambda \rangle$ on $1/B^*$ for all aspect ratios. At high $1/B^*$, the film wetting correction also collapses the data to a common curve for all $w/b > 58.4$, though a slight displacement downward of the large w/b data remains.

While our $\langle \lambda \rangle$ results do not exhibit the classical scaling with $1/B^*$, the maximum value of the fluctuating finger width observed during finger evolution, λ_{max} , does, as shown in Fig. 3(c). (λ_{max} is the largest width value observed over time in a region behind the tip; observing only near the tip excludes widening due to relaxation effects.) λ_{max} does not exhibit a peak with decreasing $1/B$, and while λ_{max} has more statistical noise than $\langle \lambda \rangle$, the λ_{max} data collapse onto similar, monotonically decreasing curves that agree with the McLean-Saffman prediction at low $1/B^*$. The λ_{max} data fall below the McLean-Saffman curve at high $1/B^*$, but not as much as the data for $\langle \lambda \rangle$. These λ_{max} data suggest that the fluctuations represent an intermittent narrowing of the fingers from their “ideal” width.

The finger width fluctuations and the peak in $\langle \lambda \rangle$ vs $1/B$ have proven robust under variations of experimental conditions. Both high and low viscosity oils gave the same results for the same geometric configuration. By treating the stainless steel spacers with an anti-wetting agent, we changed the contact angle where the interface is pinned at the back of the channel; this difference can be seen between Figs. 1(b) and 1(c); we again found the same results for the same geometric configuration. To ensure that the syringe pump was not imposing the observed velocity fluctuations, we pumped one data set by gravity siphoning, again with identical results. The effect of variations in the gap between the plates was examined for several aspect ratios by overclamping the channel along the sides, which increases the intrinsic gap variations by a factor of 2.5 (measured interferometrically); both the finger width fluctuations and the location of $\langle \lambda \rangle_{\text{peak}}$ with respect to $1/B^*$ were unchanged, although $\langle \lambda \rangle$ values decreased by about 4% near the peak.

Though the fluctuation power law we observed remained unchanged for *all* experimental variations, we did discover that measurements on two channels with $w/b=394$ but different values of w and b yielded different results. The $\langle \lambda \rangle$ values for both setups departed the common curve at approximately the same point with decreasing $1/B^*$, but at lower $1/B^*$ the behavior was different; compare 394W (394 wide) with 394N (394 narrow) in Fig. 3(b). This difference suggests that either the width for a given w/b and Ca is not unique, or perhaps a third parameter, as yet unknown, is necessary to describe the problem for high w/b and/or low Ca . Consistent with this are the $\langle \lambda \rangle$ data for $w/b=247$ and

314, which superpose closely even though they have no geometric parameters in common.

It is unlikely that the fluctuations are caused by an instability of the film wetting layer because the film is very thin at low capillary numbers, where the fluctuations are largest. Film wetting fluctuations would also cause deviations in the growth rate of the area of the fingers, which we do not observe. We speculate that the fluctuations in finger width may be a consequence of long-time relaxations of the interface at the back of the channel, observable particularly in Fig. 1(c). We also speculate that the peak in $\langle \lambda \rangle$ observed for decreasing $1/B$ at large aspect ratios may also occur at small aspect ratios, but at values of $1/B$ too small to be reached.

In conclusion, we have discovered fluctuations that intermittently narrow evolving single fingers; the magnitude of these fluctuations follows a power law with the capillary

number for all aspect ratios studied. We also have found a departure from the classic scaling of finger width versus $1/B$ for large aspect ratios ($w/b \geq 250$); the average finger width narrows at low $1/B$, while the maximum finger width increases. These phenomena are not predicted by existing viscous fingering theories even though the phenomena are most pronounced for parameters that more closely match the theoretical assumptions.

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